SOME PROBLEMS IN SEQUENTIAL SIMULATION WITH SELF-SIMILAR PROCESSES *

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ABSTRACT

It is generally accepted that self-similar processes may provide better models for teletraffic in modern telecommunication networks than Poisson processes. If stochastic self-similarity of teletraffic is not taken into account, it can lead to inaccurate conclusions about performance of networks. Thus, an important requirement for conducting simulation studies of networks is the ability to generate long synthetic self-similar sequences of incremental processes, to transform them into inter-event time intervals, and to do this accurately and quickly. A case study is discussed to show how many observations and how much time are needed in steady-state simulation of queueing models with self-similar input processes. This is compared with simulation run lengths of the same queueing models fed by Poisson processes.

1 INTRODUCTION

The search for accurate mathematical models of data streams in modern telecommunication networks has attracted a considerable amount of interest in the last few years. The reason is that several recent teletraffic studies of local and wide area networks, including the world wide web, have shown that commonly used teletraffic models, based on Poisson or related processes, are not able to capture the self-similar (or fractal) nature of teletraffic (Leland et al. 1994; Likhanov et al. 1995; Paxson and Floyd 1995; Ryu 1996), especially when they are engaged in such sophisticated services as variable-bit-rate (VBR) video transmission (Garrett and Willinger 1994; Krunz and Makowski 1998; Rose 1997). The properties of teletraffic in such scenarios are very different from both the properties of conventional models of telephone traffic and the traditional models of data traffic generated by computers.

The use of traditional models of teletraffic can result in overly optimistic estimates of performance of telecommunication networks, insufficient allocation of communication and data processing resources, and difficulties in ensuring the quality of service expected by network users (Beran 1992; Neidhardt and Wang 1998; Paxson and Floyd 1995). On the other hand, if the strongly correlated character of teletraffic is explicitly taken into account, this can also lead to more efficient traffic control mechanisms.

Several methods for generating pseudo-random self-similar sequences have been proposed. They include methods based on fractional Gaussian noise (Mandelbrot 1971), fractional ARIMA processes (Hosking 1984), the $M/G/\infty$ queue model (Krunz and Makowski 1998; Leland et al. 1994), autoregressive processes (Cario and Nelson 1998; Granger 1980), spatial renewal processes (Taralp et al. 1998), etc. Some of them generate asymptotically self-similar sequences and require large amounts of CPU time. For example, Hosking's method (Hosking 1984), based on the F-ARIMA$(0, d, 0)$ process, needs many hours to produce a self-similar sequence with 131,072 (2^{17}) numbers on a Sun SPARCstation 4 (Leland et al. 1994). It requires $O(n^2)$ computations to generate n numbers. Even though exact methods of generation of self-similar sequences exist (for example: (Mandelbrot 1971)), they are only fast enough for short sequences.

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They are usually inappropriate for generating long sequences because they require multiple passes along generated sequences. To overcome this, approximate methods for generation of self-similar sequences in simulation studies of telecommunication networks have also been proposed (Lau et al. 1995; Paxson 1997).

An important requirement for conducting simulation studies of telecommunication networks is the ability to generate long synthetic self-similar sequences of incremental processes, long enough to ensure arbitrary statistical precision of the final simulation results. Very little is known on appropriateness of selection of specific inter-arrival processes and production of inter-event time intervals for arrival counts generated by an FGN process still remains an open research issue (Jeong et al. 1999b; Leroux and Hassan 1999; Paxson 1997).

Three methods for generation of self-similar sequences were considered in (Jeong et al. 1999). In this paper we study an additional method, based on *Fractional Gaussian Noise* and *Daubechies Wavelets* (*FGN-DW*) which has promising properties, and consider the transformation of count processes into inter-arrival processes. This generator was proposed in (Jeong et al. 1999a). Then, the inter-arrival processes are used in steady-state simulation of queueing models with self-similar input processes.

A summary of the basic properties of self-similar processes is given in the next section. Generation and transformations of the *FGN-DW* count processes are described in Section 3. In Section 4, a case study is discussed to show how many observations and how much time are needed in steady-state simulation of queueing models with self-similar input processes (called $SSM/M/1/\infty$). This is compared with simulation run lengths of the same queueing models fed by short-range dependence (SRD) processes such as $M/M/1/\infty$ queueing system. The influence of the Hurst parameter on simulation run-length is also analysed, before the final conclusions are formulated.

2 SELF-SIMILAR PROCESSES AND THEIR PROPERTIES

Basic definitions of self-similar processes are as follows: A continuous-time stochastic process $\{X_t\}$ is strongly *self-similar* with a self-similarity parameter H $(0 < H < 1)$, known as the Hurst parameter, if for any positive stretching factor c, the rescaled process with time scale $ct, c^{-H}X_{ct}$, is equal in distribution to the original process $\{X_t\}$ (Beran 1994). This means that, for any sequence of time points $t_1, t_2, \ldots, t_n$, and for all $c > 0$, $\{c^{-H}X_{ct_1}, c^{-H}X_{ct_2}, \ldots, c^{-H}X_{ct_n}\}$ has the same distribution as $\{X_{t_1}, X_{t_2}, \ldots, X_{t_n}\}$.

In discrete-time case, let $\{X_k\} = \{X_k : k = 0, 1, 2, \ldots\}$ be a (discrete-time) stationary process with mean μ, variance σ^2, and autocorrelation function (ACF) $\{\rho_k\}$, for $k = 0, 1, 2, \ldots$, and let $\{X_k^{(m)}\}_{k=1}^{\infty} = \{X_1^{(m)}, X_2^{(m)}, \ldots\}$, $m = 1, 2, 3, \ldots$, be a sequence of batch means, i.e., $X_k^{(m)} = (X_{km-m+1} + \ldots + X_{km})/m, k \geq 1$.

The process $\{X_k\}$ with $\rho_k \to k^{-\beta}$, as $k \to \infty, 0 < \beta < 1$, is called *exactly self-similar* with $H = 1 - (\beta/2)$, if $\rho_k^{(m)} = \rho_k$, for any $m = 1, 2, 3, \ldots$. In other words, the process $\{X_k\}$ and the averaged processes $\{X_k^{(m)}\}$, $m \geq 1$, have identical correlation structure. The process $\{X_k\}$ is *asymptotically self-similar* with $H = 1 - (\beta/2)$, if $\rho_k^{(m)} \to \rho_k$, as $m \to \infty$.

The most frequently studied models of self-similar traffic belong either to the class of fractional autoregressive integrated moving-average (F-ARIMA) processes or to the class of fractional Gaussian noise processes; see (Hosking 1984; Leland et al. 1994; Paxson 1997). F-ARIMA(p, d, q) processes were introduced by Hosking (Hosking 1984) who showed that they are asymptotically self-similar with Hurst parameter $H = d + \frac{1}{2}$, as long as $0 < d < \frac{1}{2}$. On the other hand, the incremental process $\{Y_k\} = \{X_k - X_{k-1}\}, k \geq 0$, is called the *fractional Gaussian noise* (FGN) process, if $\{X_k\}$ represents a fractional Brownian motion (FBM) random process. This process is a (discrete-time) stationary Gaussian process with mean μ, variance σ^2 and $\{\rho_k\} = \{\frac{1}{2}(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H})\}, \quad k > 0$. A FBM process is characterised by three properties (Mandelbrot and Wallis 1969): (i) it is a continuous zero-mean Gaussian process $\{X_t\} = \{X_s : s \geq 0$ and $\quad 0 < H < 1\}$ with ACF given by $\{\rho_{s,t}\} = \{\frac{1}{2}(s^{2H} + t^{2H} - |s-t|^{2H})\}$, where s is time lag and t is time; (ii) its increments $\{X_t - X_{t-1}\}$ form a stationary random process; (iii) it is self-similar with Hurst parameter H, that is, for all $c > 0$, $\{X_{ct}\} = \{c^H X_t\}$, in the sense that, if time is changed by the ratio c, then $\{X_t\}$ is changed by c^H.

Main properties of self-similar processes include:

- *Slowly decaying variance*. The variance of the sample mean decreases more slowly than the reciprocal of the sample size, that is, $Var[\{X_k^{(m)}\}] \to c_1 m^{-\beta_1}$ as $m \to \infty$, where c_1 is a constant and $0 < \beta_1 < 1$.

- *Long-range dependence*. A process $\{X_k\}$ is called a stationary process with *long-range dependence (LRD)* if its ACF $\{\rho_k\}$ is non-summable, that is, $\sum_{k=0}^{\infty} \rho_k = \infty$. The speed of decay of autocorrelations is more hyperbolic than exponential.

- *Hurst effect.* Self-similarity manifests itself by a straight line of slope β_2 on a log-log plot of the R/S statistic. For a given set of numbers $\{X_1, X_2, \ldots, X_n\}$ with sample mean $\hat{\mu} = E\{X_i\}$ and sample variance $S^2(n) = E\{(X_i - \hat{\mu})^2\}$, Hurst parameter H is presented by the *rescaled adjusted range* $\frac{R(n)}{S(n)}$ (or R/S *statistic*) where $R(n) = \max\{\sum_{i=1}^{k}(X_i - \hat{\mu}), 1 \leq k \leq n\} - \min\{\sum_{i=1}^{k}(X_i - \hat{\mu}), 1 \leq k \leq n\}$ and S is estimated by $S(n) = \sqrt{E\{(X_i - \hat{\mu})^2\}}$. Hurst found empirically that for many time series observed in nature, the expected value of $\frac{R(n)}{S(n)}$ asymptotically satisfies the power-law relation: $E[\frac{R(n)}{S(n)}] \rightarrow c_2 n^H$, as $n \rightarrow \infty$, with $0.5 < H < 1$, where c_2 is a finite positive constant. The Hurst parameter H is typically about 0.73, observations X_k from SRD processes are known to satisfy $E[\frac{R(n)}{S(n)}] \rightarrow c_3 n^{0.5}$ as $n \rightarrow \infty$, with c_3 a finite positive constant. This discrepancy of the H values obtained by LRD and SRD processes, i.e., $H = 0.73$ and $H = 0.5$, respectively, is generally referred to as the *Hurst effect*; see for example (Beran 1994; Cox 1984; Leland et al. 1994).
- $1/f$*-noise.* The spectral density $f(\lambda; H)$ obeys a power law near the origin, i.e., $f(\lambda; H) \rightarrow c_4 \lambda^{1-2H}$, as $\lambda \rightarrow 0$, where c_4 is a finite positive constant and $0.5 < H < 1$.

3 TELETRAFFIC GENERATION AND SIMULATION

We claim that the *FGN-DW* transformation is sufficiently fast for generation of synthetic self-similar sequences, to be used as simulation input data (Jeong et al. 1999a). Let us briefly introduce the *FGN-DW* method itself.

3.1 Generation of Self-Similar Teletraffic Using FGN-DW

The general strategy behind our method is the same as in (Paxson 1997). The algorithm consists of the following steps (for more detailed discussions, see (Jeong et al. 1999a)):

Step.1 Calculate a sequence of values $\{f_1, f_2, \cdots, f_n\}$ using $f(\lambda; H) = c_f|\lambda|^{1-2H} + O(|\lambda|^{min(3-2H,2)})$ where $c_f = \sigma^2(2\pi)^{-1}sin(\pi H)\Gamma(2H+1)$ and $O(\cdot)$ represents the residual error, $f_i = \hat{f}(\frac{\pi i}{n})$, corresponding to the spectral density of an FGN process for frequencies from $\frac{\pi}{n}$ to π.

Step.2 Adjust $\{f_i\}$ by multiplying them by realisations of an independent exponential random variable with mean equal 1.

Step.3 Generate a sequence $\{x_1, x_2, \cdots, x_n\}$ of complex numbers such that $|x_i| = \sqrt{\hat{f}_i}$ and the phase of x_i is uniformly distributed between 0 and 2π. The random phase technique, taken from (Schiff 1992), preserves the distribution of spectral density of $\{\hat{f}_i\}$, but ensures that different sequences generated using the method will be independent. It also makes the marginal distribution of the final sequence normal. The phase randomisation makes the different frequency components independent (Paxson 1997).

Step.4 Calculate two synthetic coefficients of orthonormal Daubechies wavelets which are used in the inverse discrete wavelet transform (IDWT) (Nartallo et al. 1996). Then, generate the approximately self-similar FGN sequence in time domain by using the IDWT from $\{x_i\}$. We use the Daubechies wavelets (Daubechies 1992) because it produces more accurate coefficients of wavelets than Haar wavelets (Wickerhauser 1994).

Using the above steps, the proposed *FGN-DW* method gives a fast generator of well approximated self-similar sequences of numbers representing FGN processes which can be interpreted as differential arrival rates.

3.2 Transformation of Self-Similar Count Processes into Inter-Arrival Processes

Simulation studies of telecommunication networks require generation of arbitrarily long sequences of inter-arrival times of transmitted data packets. Therefore, a mechanism is needed to transform self-similar processes representing the arrival counts, or differential arrival rates (see the above subsection.), into suitable sequences of inter-arrival times of packets, while preserving appropriate characteristics (Jeong et al. 1999b). In this paper, we use a count process generated by *FGN-DW* to obtain exponential inter-arrival distribution.

Let $F_Y(y)$ be a marginal cumulative density function (CDF) corresponding to a process $\{Y_k\}$. Then the process $\{Y_k\}$ with the desirable marginal CDF $F_Y(y)$ is produced from the process $\{X_k\}$ by using the following transformation. Given a self-similar sequence of the *FGN-DW* process $\{X_k\}$, we can transform the marginal distribution by mapping each number as $\{Y_k\} = F_Y^{-1}(F_N(X_k)), \quad k = 1, 2, \ldots$.

In order to estimate the CDF, we use that from Cody 1969. Rational approximations for the complementary error functions erfc (x), with maximal relative errors ranging down to between 6 x 10^{-19} and 3 x 10^{-20} were proposed by (Cody 1969).

The CDF of the exponential distribution is $F_X(x) = 1 - e^{-\lambda x}$. To generate exponential variates, the inverse transformation is defined as: $\{Y_k\} = -(1/\lambda) * \log(F_N(X_k))$.

4 NUMERICAL RESULTS

We look at how many observations and how much time are needed in sequential steady-state simulation of a queueing system with self-similar input processes. An $M/M/1/\infty$ and $SSM/M/1/\infty$ queueing systems have been simulated in the method based on spectral analysis when one simulation engine is used in AKAROA-2 (Ewing et al. 1999), a fully automated tool of sequential simulation designed for stopping simulation with a pre-specified statistical precision of the final results.

Figure 1 shows the number of theoretically required observations, and the mean number of empirical observations need for stopping simulation with the required statistical precision, for different values of traffic intensity ρ, when analysing steady-state mean response time in an $M/M/1/\infty$ queueing system. Each mean experimental run length of simulation is an average over 30 replications. Confidence intervals of relative half-widths of 10% or less, at 0.95 confidence level, are also shown in Figure 1. While for $\rho \leq 0.6$ the mean number of empirical observations is slightly higher than the number of theoretically required observations, for $0.6 < \rho \leq 0.9$ the mean number of empirical observations is lower than the number of observations required theoretically. Both numbers quickly grow as $\rho \to 1$. The range of the reported mean numbers of empirical observations needed is between 1787 and 128211.

The results in Figure 1 show that, analysis of an $SSM/M/1/\infty$ queueing system with self-similar input processes requires many more observations than an $M/M/1/\infty$ queueing system. The mean number of observations, or simulation run length, significantly increases with H of the input stream. For $\rho = 0.4$ and each H = 0.6, 0.7 and 0.8, the mean number of empirical observations for the $SSM/M/1/\infty$ queueing system is 4484 (13.4%), 13540 (242.4%) and 233023 (5791.9%), respectively, while for $\rho = 0.4$, the $M/M/1/\infty$ queueing system needs 3955 observations (($\cdot$ %) are the relative increases of the run-length against the $M/M/1/\infty$ queueing system). For $\rho = 0.8$ and each H = 0.6 and 0.7, the mean number of empirical observations for the $SSM/M/1/\infty$ queueing system is 76373 (228.4%) and 539781 (2221.2%), respectively, while for $\rho = 0.8$, the $M/M/1/\infty$ queueing system only needs 23524 observations.

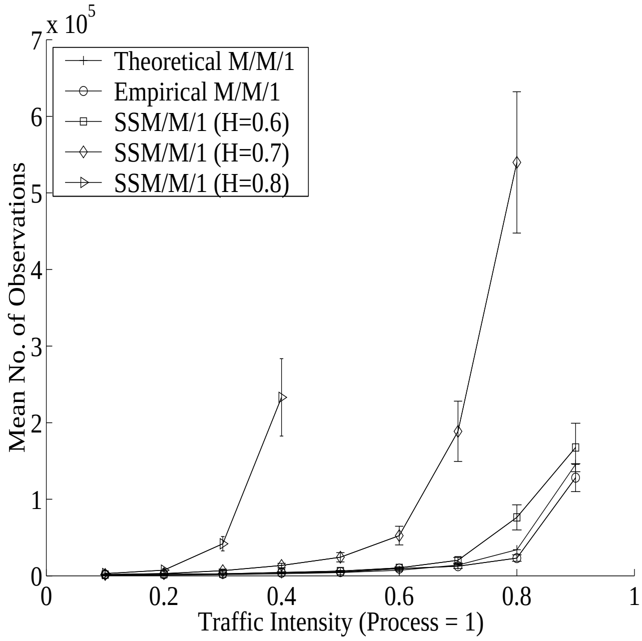

Figure 1: Mean numbers of observations needed in sequential analysis of steady-state mean response time: in an $M/M/1/\infty$ and $SSM/M/1/\infty$ queueing system with relative precision $\leq$ 10%, 95% CI when one simulation engine is used.

Figure 2 shows the mean response time needed in the sequential analysis of a steady-state simulation in an $SSM/M/1/\infty$ and an $M/M/1/\infty$ queueing system with a relative statistical error $\leq$ 10%, 95% CI. Even though $\rho = 0.4$, for $H = 0.6, 0.7$ and 0.8, the mean response time of the $SSM/M/1/\infty$ is 0.174 (4.8%), 0.206 (24.1%), and 0.3291 (98.3%) seconds, respectively, but the mean response time of the $M/M/1/\infty$ queueing system is 0.166 seconds. For $\rho = 0.8$ and $H = 0.6$ and 0.7, the mean response times of the $SSM/M/1/\infty$ queueing system are 0.6722 (35.5%) and 2.086 (320.6%) seconds, respectively, while for $\rho = 0.8$, the mean response time of the $M/M/1/\infty$ queueing system is only 0.496 seconds.

Therefore, the mean response time of the $SSM/M/1/\infty$ queueing system in Figure 2 rapidly increases as H increases, while the mean response time of the $M/M/1/\infty$ queueing system slowly increases.

Thus, the results tell us that performance of queueing systems under Poisson processes and self-similar processes are very different. If self-similar processes are taken into account in simulation studies of telecommunication networks, they will give more realistic results than Poisson processes. We may also face difficulty in sequential steady-state simulation of a queueing system when the H value is especially very high, since methods of long bulk generation with self-similar processes require a large amount of time.

5 CONCLUSIONS

We have examined the main issue of stochastic simulation of telecommunication networks with self-similar teletraffic. We looked at how many observations and how much time are needed in sequential steady-state simulation of a queueing system with self-similar input processes. As we have shown, assuming self-similar inter-arrival processes, one needs many more observations to obtain the final simulation results with a required precision than when assuming Poisson inter-arrival processes. To secure a predefined statistical precision of simulation final results, one would need to generate arbitrary long sequences of inter-arrival times, and this would need to be done sequentially. At this stage, an algorithm for sequential generation of self-similar sequences has not been proposed.

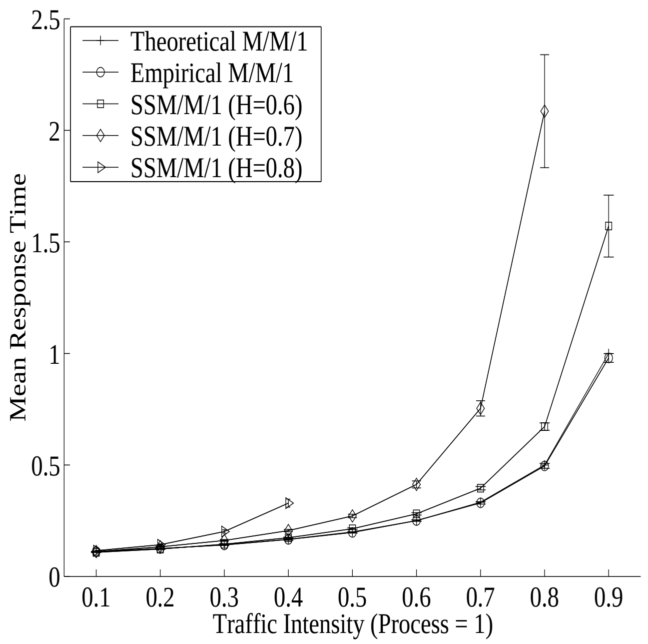

Figure 2: Mean response time needed in sequential analysis of steady-state mean response time: in an $M/M/1/\infty$ and $SSM/M/1/\infty$ queueing system with relative precision $\leq 10\%$, 95% CI when one simulation engine is used.

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REFERENCES

Beran, J. 1992. "Statistical Methods for Data with Long Range Dependence." *Statistical Science*, 7, no.4: 404-427.

Beran, J. 1994. *Statistics for Long-Memory Processes.* Chapman and Hall, New York.

Cario, M.C. and B.L. Nelson. 1998. "Numerical Methods for Fitting and Simulating Autoregressive-to-Anything Processes." *INFORMS Journal on Computing*, 10, no.1: 72-81.

Cody, W.J. 1969. "Rational Chebyshev Approximations for the Error Function." *Mathematics of Computation*, 23, no. 107: 631-637.

Cox, D.R. 1984. "Long-Range Dependence: a Review." *In Statistics: An Appraisal.* Iowa State Statistical Library, The Iowa State University Press, 55-74.

Daubechies, I. 1992. *Ten Lectures on Wavelets.* CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM Press, Philadelphia, Pennsylvania, 61.

Ewing, G.; K. Pawlikowski; and D. McNickle. 1999. "AKAROA-2: Exploiting Network Computing by Distributing Stochastic Simulation." In *Proceedings of 13th European Simulation Multiconference, ESM'99* (Warsaw, Poland), 1: 175-181.

Garrett, M.W. and W. Willinger. 1994. "Analysis, Modeling and Generation of Self-Similar VBR Video Traffic." In *Computer Communication Review, Proceedings of ACM SIGCOMM'94* (London, UK, Aug.), 24, no. 4: 269-280.

Granger, C.W.J. 1980. "Long Memory Relationships and the Aggregation of Dynamic Models." *Journal of Econometrics*, 14: 227-238.

Hosking, J.R.M. 1984. "Modeling Persistence in Hydrological Time Series Using Fractional Differencing." *Water Resources Research*, 20, no. 12 (Dec.): 1898-1908.

Jeong, H.-D.J.; D. McNickle; and K. Pawlikowski. 1999. "A Comparative Study of Three Self-Similar Teletraffic Generators." In *Proceedings of 13th European Sim-*

ulation Multiconference, ESM'99 (Warsaw, Poland), 1: 356-362.

Jeong, H.-D.J.; D. McNickle; and K. Pawlikowski. 1999a. "Fast Self-Similar Teletraffic Generation Based on FGN and Wavelets." In *Proceedings of the IEEE International Conference on Networks, ICON'99* (Brisbane, Australia), 75-82.

Jeong, H.-D.J.; D. McNickle; and K. Pawlikowski. 1999b. "Generation of Self-Similar Time Series for Simulation Studies of Telecommunication Networks." In *Proceedings of the First Western Pacific and Third Australia-Japan Workshop on Stochastic Models in Engineering, Technology and Management* (Christchurch, New Zealand), 221-230.

Krunz, M. and A. Makowski. 1998. "A Source Model for VBR Video Traffic Based on $M/G/\infty$ Input Processes." In *Proceedings of IEEE INFOCOM'98* (San Francisco, CA, USA, Mar.), 1441-1448.

Lau, W-C.; A. Erramilli; J.L. Wang; and W. Willinger. 1995. "Self-Similar Traffic Generation: the Random Midpoint Displacement Algorithm and Its Properties." In *Proceedings of IEEE ICC'95* (Seattle, WA), 466-472.

Leland, W.E.; M.S. Taqqu; W. Willinger; and D.V. Wilson. 1994. "On the Self-Similar Nature of Ethernet Traffic (Extended Version)." *IEEE/ACM Transactions on Networking*, 2, no. 1: 1-15.

Likhanov, N.; B. Tsybakov; and N.D. Georganas. 1995. "Analysis of an ATM Buffer with Self-Similar ("Fractal") Input Traffic." In *Proceedings of IEEE INFOCOM'95, 985-992.*

Leroux, H. and M. Hassan. 1999. "Generating Packet Inter-Arrival Times for FGN Arrival Processes." In *The 3rd New Zealand ATM and Broadband Workshop* (Hamilton, New Zealand), 1-10.

Mandelbrot, B.B. 1971. "A Fast Fractional Gaussian Noise Generator." *Water Resources Research*, 7: 543-553.

Mandelbrot, B.B. and J.R. Wallis. 1969. "Computer Experiments with Fractional Gaussian Noises." *Water Resources Research*, 5, no. 1: 228-267.

Nartallo, C.M.; N.G. Prelcic; S.J.G. Galan; and C.S. Cabanelas. 1996. "Wavelet Toolbox for Matlab." Available through http: //www.tsc.uvigo.es/~wavelets/uvi_wave.html.

Neidhardt, A.L. and J.L. Wang. 1998. "The Concept of Relevant Time Scales and Its Application to Queueing Analysis of Self-Similar Traffic (or Is Hurst Naughty or Nice?)." In *Proceedings ACM SIGMETRICS'98* (Madison, Wisconsin, USA, Jun.), 222-232.

Paxson, V. 1997. "Fast, Approximate Synthesis of Fractional Gaussian Noise for Generating Self-Similar Network Traffic." *ACM SIGCOMM, Computer Communication Review*, 27, no. 5: 5-18.

Paxson, V. and S. Floyd. 1995. "Wide-Area Traffic: the Failure of Poisson Modeling." *IEEE/ACM Transactions on Networking*, 3, no. 3 (Jun.): 226-244.

Rose, O. 1997. *Traffic Modeling of Variable Bit Rate MPEG Video and Its Impacts on ATM Networks.* PhD thesis, Bayerische Julius-Maximilians-Universität Würzburg.

Ryu, B.K. 1996. *Fractal Network Traffic: from Understanding to Implications.* PhD thesis, Graduate School of Arts and Science, Columbia University.

Schiff, S. J. 1992. "Resolving Time-Series Structure with a Controlled Wavelet Transform." *Optical Engineering*, 31, no. 11: 2492-2495.

Taralp, T.; M. Devetsikiotis; and I. Lambadaris . 1998.
"Efficient Fractional Gaussian Noise Generation Using the Spatial Renewal Process." In *Proceedings IEEE International Communications Conference (ICC'98)* (Atlanta, GA, USA, Jun.), S41.3.1-S41.3.5.

Wickerhauser, M.V. 1994. *Adapted Wavelet Analysis from Theory to Software.* A K Peters, Ltd., Wellesley, Massachusetts.

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