Quality of Sequential Regenerative Simulation

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Abstract

Regenerative simulation (RS) is a method of stochastic steady-state simulation in which output data are collected and analysed within regenerative cycles (RCs). Since data collected during consecutive RCs are independent and identically distributed, there is no problem with the initial transient period in simulated processes, which is a perennial issue of concern in all other types of steady-state simulation. In this paper, we address the issue of experimental analysis of the quality of sequential regenerative simulation in the sense of the coverage of the final confidence intervals of mean values. The ultimate purpose of this study is to determine the best version of RS to be implemented in Akaroa2 [1], a fully automated controller of distributed stochastic simulation in computer network environments.

Keywords : regenerative simulation, sequential steady-state simulation, coverage analysis

1 Introduction

Sequential statistical analysis of output data in stochastic simulation, used for controlling the length of simulation, is regarded as the only practical way of securing appropriate level of credibility of the final simulation results [2]. Following this approach, simulation is progressing from one checkpoint to the next one, until a prespecified accuracy of all point estimators is obtained. Probably the most commonly used stopping criterion for sequential steady-state simulation is the *relative precision of confidence intervals* defined as the ratio of the half-width of the confidence interval (at a given confidence level) and the current estimate of a given estimated performance measure [3]. An experiment is stopped at the checkpoint at which the required relative precision of the final results is reached.

In non-regenerative methods of steady-state simulation output data analysis, like Spectral Analysis and Batch Means, one has to discard data collected during the initial transient periods and observe the process for a subsequent sufficiently long time period later on, to obtain satisfactorily credible estimates. Determination of the length of the initial transient period is often nontrivial and likely to require sophisticated statistical techniques [3]. Therefore, regenerative method (RM) of analysis of simulation output data is very attractive alternative, because it avoids this problem. In regenerative stochastic processes, regenerative cycles (RCs) produce batches of independent and identically distributed data, and the final precision of results depends on the number of RCs observed.

Standard sequential stopping rules of sequential simulation [3], like the *relative precision of confidence intervals* can be used also in conjunction with RM. However, RM in sequential steady-state simulation can lead to inaccurate results if the simulation experiment stops too early, when the sequential stopping criterion is accidently temporarily met. Some sequential stopping rules for RM were proposed and tested by Sauer [4] and Lavenberg and Sauer [5]. Following the stopping rule proposed in [5], the simulation should be stopped when the minimum number of RCs is observed (assumed to be 10) and the required precision is reached. In [4], it was argued that the simulation run length should be associated with some minimum simulation time. As the results of our studies show such approaches are not longer satisfactory or needed, taking into account currently available computing resources.

One of the main quality criteria used for assessing the quality of methods of simulation output data analysis in stochastic simulation is the coverage of the final confidence intervals it produces, defined as the proportion of confidence intervals which contain the true value. Such experimental confidence level should be confronted with the theoretical confidence level of the final estimates. Any good method of analysis of simulation output data should produce narrow and stable confidence intervals, and the relative frequency of such an interval containing the true value of the estimated performance measure should not differ from the assumed theoretical confidence level. In the past, coverage analyses of various sequential stopping rules for RM, including those in [4] and [5], were conducted using fixed numbers of replications (for example, 50 and 100, as [4] and [5], respectively).

But, as recently argued in [6], coverage analysis should be conducted sequentially, to secure statistically accurate results. The rules of sequential coverage analysis for non-RS (regenerative simulation) have been proposed in [6]. In this paper, an adaptation of these rules for sequential RS is presented in Section 3. This is an enhanced version of the coverage analysis, based on F distribution, which, as shown in [7], leads to more efficient interval estimators of proportions. The numerical results of coverage analysis of the sequential RM applied for estimating steady-state means, and reported in Section 4, were obtained in our quest for the most robust method of sequential analysis of simulation output data, to be implemented in Akaroa2 [1], a fully automated controller of distributed stochastic simulation on multiple networked processors, in Multiple Replications In Parallel (MRIP) scenario [8]. The results of coverage analysis of two other methods of sequential estimation of steady-state means, namely based on Non-overlapping Batch Means and Spectral Analysis (in its version originally proposed by Heidelberger and Welch [9]) were presented in [6]. The results of coverage analysis of sequential methods of estimation of steady-state quantiles are reported in [10].

Analysis of coverage is of course limited to analytically tractable systems, since the theoretical value of the parameter of interest has to be known. Because of that, it has even been claimed that there is no justification for experimental coverage analysis, since there is no theoretical basis for extrapolating results found for simple, analytically tractable systems to more complex systems, which are subjects of practical simulation studies [11]. On the other hand, no theory of coverage for finite sample sizes exists, and in this situation, experimental coverage analysis of analytically tractable systems remains the only method available for testing validity of methods proposed for simulation output analysis. Certainly nobody is ready to accept a method of simulation output data analysis showing very poor quality in experimental studies of coverage.

2 The Properties of RM

As known, RS is based on the assumption that any regenerative process starts afresh (probabilistically) at each consecutive regenerative point. Thus, observations grouped into batches of random length, determined by successive regenerative instants of the simulated process, are statistically independent, and that includes the first RC, if the simulation starts from a regenerative state.

For instance, when simulating an $M/G/1/\infty$ queueing system, any instant of time when this system reaches the state 0 (no customer present) represents a regenerative point at the boundary of two consecutive RCs. After any such instant of time, no event from the past influences the future evolution of the system. As a consequence of the independent and identically distributed output data within consecutive RCs, the problems related with the initial transient period and autocorrelations vanish [12], [13], [14], [15].

While the accuracy of the final simulation results from RS depends on the number of simulated RCs, the rate at which RCs occur depends on the simulated system. For example, in heavily loaded but stable queueing systems regenerative states can occur very rarely, making the RS very ineffective, since it becomes difficult, if possible at all, to form a reliable point estimate and confidence intervals.

As known, RM uses estimators in the form of a ratio of two variables; see for example [12]. To estimate steady-state mean EX of, for example, waiting times in a queueing system on the basis of observed waiting times $x_1, x_2, x_3, \ldots,$ of consecutive customers, we are given the pairs of (secondary) output data $(a_1, y_1), (a_2, y_2), \ldots, (a_n, y_n)$ which are realisations of i.i.d. random variables A_k and Y_k, $1 \leq k \leq n$, where A_k and Y_k denote, respectively, the number of customers processed and the sum of the waiting times in kth RC. Let $\overline{Y}(n)$, $\overline{a}(n)$, $s_{11}^2(n)$, $s_{22}^2(n)$, and $s_{12}^2(n)$ be the usual unbiased estimators for $E[Y]$, $E[A]$, $Var[Y]$, $Var[A]$, and $Cov[Y, A]$ for any i, respectively; that is $\overline{Y}(n) = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $\overline{a}(n) = \frac{1}{n}\sum_{i=1}^{n} a_i$, $s_{11}^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \overline{Y}(n)\right)^2$, $s_{22}^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}(a_i - \overline{a}(n))^2$, and $s_{12}^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \overline{Y}(n)\right)(a_i - \overline{a}(n))$.

As a consequence of the *strong law of large numbers* [12], the point estimator of the mean

$$\hat{r}(n) = \frac{\overline{Y}(n)}{\overline{a}(n)}$$

is strongly consistent estimator of steady-state mean EX; that is, $\hat{r}(n) \to EX$ as $n \to \infty$. Moreover, the estimator for variance

$$s^2(n) = \{s_{11}^2(n) - 2\hat{r}(n)s_{12}^2(n) + \hat{r}^2(n)s_{22}^2(n)\}$$

is also strongly consistent; that is, $s^2(n) \to Var(X)$ as $n \to \infty$.

A $100(1-\alpha)\%$ confidence interval for the steady-state mean obtained by applying RM is given by

$$\hat{r}(n) \pm \frac{s(n) z_{1-\alpha/2}}{\overline{a}(n)\sqrt{n}},$$

where $z_{1-\alpha/2}$ is the $(1 - \alpha/2)$ quantile of the standard normal distribution [12], [13], [14], [15].

3 Coverage Analysis for Sequential RS

In sequential RS with a stopping rule based on the *relative precision of confidence intervals*, inaccurate estimates can be obtained if the stopping criterion is accidently temporarily satisfied, having recorded an insufficient number of RCs. As a consequence of this, sensible practise is to ensure that estimates do not come from simulation experiments with too few RCs. Recognising the significance of this factor, we have adjusted stopping rules for sequential RS by ensuring that minimum of 200 RCs in a single simulation have to be observed before it is stopped.

This minimum of 200 RCs as the shortest acceptable length of sequential RS was found experimentally and can be supported by such results as those reported in Table 1, obtained during RS of M/M/1/∞ queueing system. One can see that such very short simulation runs do have very poor coverage, below 10%, for the assumed theoretical coverage of 95%. The elimination of too short simulation runs significantly improves the quality of sequential RM, as documented by the results of coverage analysis in Figures 1 and 2. These figures show the results of sequential coverage analysis of M/M/1/∞ queueing system loaded at 0.5, with and without the restriction on the minimum of 200 recorded RCs as the length of simulation. The figures also show high initial instability of coverage. This phenomenon, similar to that reported in [6], has been the main motivation behind the proposal of sequential analysis of coverage. It is clear that the coverage analysis has to be done over sufficiently large sample of data (in this case: after sequential simulation is repeated sufficiently many times).

Ideally, the confidence interval of coverage for a method of simulation output data analysis should cover the confidence level assumed for the final results [4]. In practice, this criterion is hardly met by any method of simulation output data analysis, so, making this requirement weaker, we accept the method for practical applications if the confidence interval of its coverage is sufficiently close to the confidence level assumed. However, Figures 1 and 2 show that the final coverage was far away from the required level of 0.95.

As argued in [6], this could be caused by the fact that an insufficient number of bad final confidence intervals was recorded. (As in [6], a bad confidence interval means a confidence interval that does not cover the theoretical value of the estimated parameter). Following [6], we assumed that representativeness of data for coverage analysis requires that minimum 200 bad confidence intervals have to be recorded before sequential analysis of coverage can commence. Typical convergence of coverage to its final accurate level, if too short simulation runs are discarded when minimum number of 200 bad confidence intervals are recorded, is shown in Figure 3. Again one can see that the statistical "noise" introduced by too short simulation runs should be removed before correct conclusions regarding the quality of a given method of simulation output analysis (in this case: the RM) are drawn. As shown in Figure 3, this resulted in a jump of coverage from 0.9 to 0.95. Thus, the results of coverage of RM reported in the next section were obtained sequentially, until at least 200 bad confidence intervals have been recorded and having discarded results coming from RS shorter than 200 RCs.

4 Quality of Sequential RM

In this section we present our results of coverage analysis of sequential RM. As discussed in the previous section, these results were obtained applying the sequential analysis of coverage. These results will be additionally confronted with the results obtained following previously used method of coverage analysis, based on the fixed-sample size approach.

All results for sequential RM were obtained assuming the required precision of the final result 5% or less, at the confidence level of 0.95. The same stopping criterion applied in our sequential coverage analysis. Additionally, only simulation runs of minimum 200 RCs were taken into account, and the interval estimator of coverage was based on F distribution to ensure that the sequential analysis of coverage does not last excessively long [7].

The results of coverage reported in this section were obtained on the basis of simulation of $M/M/1/\infty$ queueing systems. The results of coverage of the sequential RM obtained from non-sequential analysis are presented in Figure 4, while Figure 5 show the same results obtained sequentially. One can clearly see that the sequential coverage analysis, with filtering off too short simulation runs and requiring recording of at least 200 bad confidence intervals, produces better (more reliable, as we have argued) results.

Generally, our results show that the sequential RM used for analysis of steady-state means can be considered as a good candidate for being implemented in such simulation packages as Akaroa2, where whole process of simulation output data is conducted automatically during simulation. Before the final recommendation is done, one should conduct full study of coverage of this method of simulation output analysis by including wider spectrum of its applications, over a range of standard stochastic systems and processes.

5 Conclusions

In this paper we formulate the rules of sequential coverage analysis for methods of output analysis used in RS. These rules have been applied in coverage analysis of the sequential RM used for estimation of steady-state means. Sequential run length control of stochastic simulation is the only efficient way for securing precision of the final simulation results.

Our initial results, obtained when using $M/M/1/\infty$ queueing systems used as the reference model, indicate the RM in its sequential version is an attractive solution for practitioners if special care is taken for avoiding too short simulation runs. Our coverage analysis of this RM is continued by studying its applications over a broader spectrum of simulation reference models. On the other hand, additional problems have to be solved before this method can be offered in fully automated simulation tools as Akaroa2. These include rules for determination of (approximate) regenerative points.

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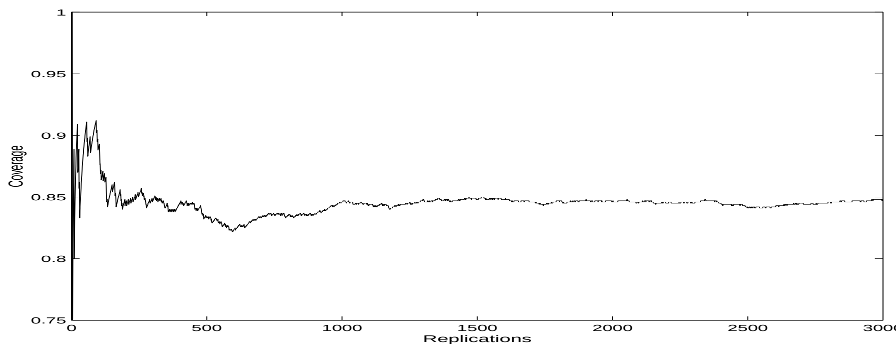

Figure 1: Convergence of coverage analysis for sequential RM with no restriction on the minimum run length (M/M/1/∞, load = 0.5).

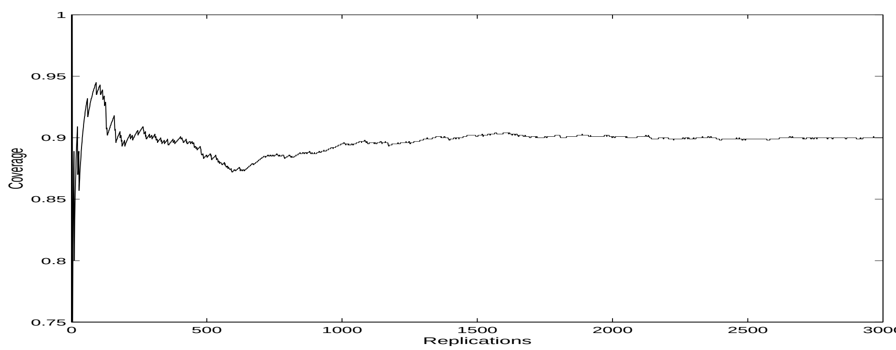

Figure 2: Convergence of coverage analysis for sequential RM with the minimum length of 200 RCs before stopping (M/M/1/∞, load = 0.5).

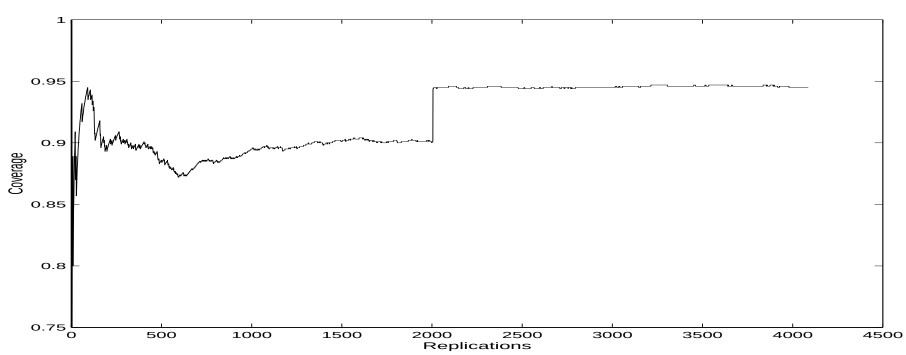

Figure 3: Convergence of coverage analysis for sequential RM with the minimum length of 200 RCs, and 200 bad confidence intervals (M/M/1/∞, load = 0.5).

Table 1: The number of too short simulation runs (less than 200 RCs) in 3000 simulation replications and their coverage (M/M/1/∞, theoretical confidence level = 0.95).

Load	*Number of too short runs*	*Coverage*
0.1	158	6.3%
0.2	167	5.4%
0.3	159	4.4%
0.4	156	5.8%
0.5	166	3.6%
0.6	159	3.1%
0.7	191	4.7%
0.8	281	3.6%
0.9	450	6.0%

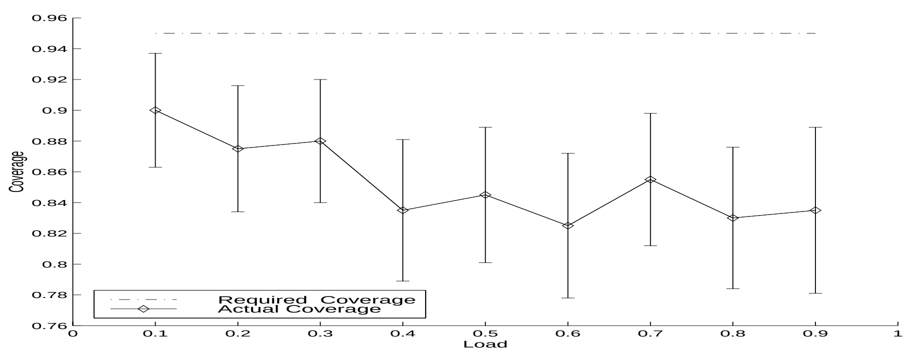

Figure 4: Non-sequential coverage analysis of sequential RM (200 replications; M/M/1/∞).

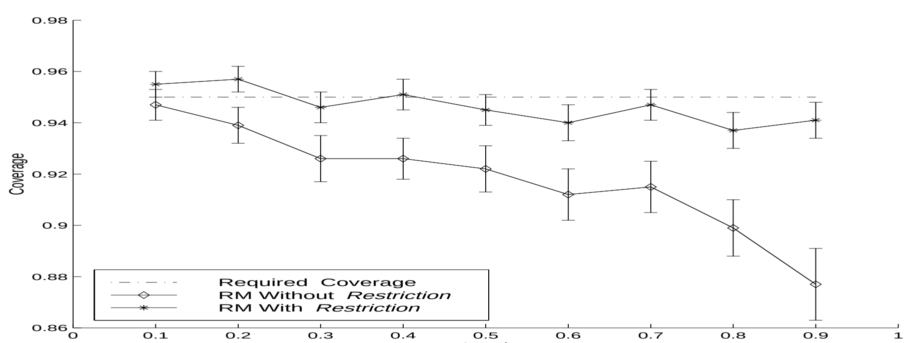

Figure 5: Sequential coverage analysis of sequential RM without and with the restriction on the minimum run length and the number of bad confidence intervals (M/M/1/∞).